Ch2 Assignment

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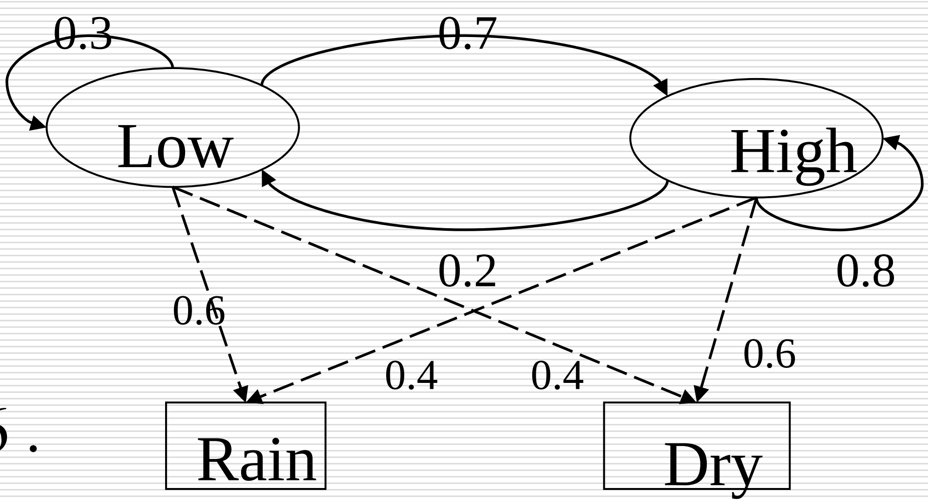
Assignment

Given the observations is "Dry","Rain","Rain", use Viterbi algorithm to predict the atmosphere pressure.

What we know:

1. Two states : 'Low' and 'High' atmospheric pressure.
2. Two observations : 'Rain' and 'Dry'.
3. Transition probabilities: P('Low'|'Low')=0.3 , P('High'|'Low')=0.7 , P('Low'|'High')=0.2, P('High'|'High')=0.8.
4. Observation probabilities :
   P('Rain'|'Low')=0.6 ,
   P('Dry'|'Low')=0.4 ,
   P('Rain'|'High')=0.4 ,
   P ('Dry'|'High')=0.6 .
5. Initial probabilities:
   say P('Low')=0.4 , P('High')=0.6 .



Please write it by hand, and turn it in within 2 weeks.

Hidden Markov models.

- Set of states: $\{s_1, s_2, \ldots, s_N\}$
- Process moves from one state to another generating a sequence of states : $s_{i1}, s_{i2}, \ldots, s_{ik}, \ldots$
- Markov chain property: probability of each subsequent state depends only on what was the previous state:

$$P(s_{ik} \mid s_{i1}, s_{i2}, \ldots, s_{ik-1}) = P(s_{ik} \mid s_{ik-1})$$

- States are not visible, but each state randomly generates one of T observations (or visible states) $\{o_1, o_2, \ldots, o_T\}$
- To define hidden Markov model, the following probabilities have to be specified:
  - matrix of transition probabilities A=(a_{ij}), a_{ij}= P(s_i | s_j)
  - matrix of observation probabilities B=($b_i(o_t)$), $b_i(o_t)$ = P(o_t | s_i) , t=1,2,…,T.
  - initial probabilities $\pi=(\pi_i)$, $\pi_i = P(s_i)$. Model is represented by $\lambda=(A, B, \pi)$.

Viterbi Algorithm

- Initialization: $\delta_1(i) = \pi_i b_i(o_1) \quad 1 \le i \le N$
- Induction:

$$\delta_t(j) = \left[\max_{1\le i\le N} \delta_{t-1}(i)\, a_{ij}\right] b_j(o_t)$$

$$\psi_t(j) = \left[\arg\max_{1\le i\le N} \delta_{t-1}(i)\, a_{ij}\right] \quad 2 \le t \le T, 1 \le j \le N$$

- Termination: $p^* = \max_{1\le i\le N} \delta_T(i) \qquad q_T^* = \arg\max_{1\le i\le N} \delta_T(i)$
- Read out path: $q_t^* = \psi_{t+1}(q_{t+1}^*) \quad t = T-1,\ldots,1$